Commutative Algebra
Final Examination

November 19 2018

This exam is of **50 marks**. Please **read all the questions carefully** and **do not cheat**. Please feel free to use whatever theorems you have learned in class after stating them clearly. Please **hand in your phones** at the beginning of the class.

1. Let A be a ring and $A[X]$ the polynomial ring. If $A[X]$ is Noetherian, then is A Noetherian? Prove your answer or give a counterexample. (5)

2. Give an example of a non-Noetherian ring. (5)

3. Let A be a domain which is integrally closed in its field of fractions. Does there exist an integral A-algebra which is a domain but is not integrally closed in its field of fractions? Prove there does not or give an example. (5)

4. If B_1 and B_2 are integral over A, is $B_1 \times B_2$ integral over A? Prove your answer or give a counterexample. (5)

5. Let A be a Noetherian ring. Show that the following are equivalent. (5)

- A is Artinian
- Spec(A) is discrete.
- $\mathsf{Spec}(A)$ is finite and discrete.

6. Give an example of a 1-dimensional Noetherian ring B in which not every fractional ideal is invertible. (5)

7. Give an example of an ideal in the ring B above which is not invertible. (5)

8. Is every 1-dimensional Noetherian local ring a discrete valuation ring? Prove your answer or give a counterexample. (5)

9. If A is Noetherian prove that $A[[X]]$, the power-series ring, is Noetherian. (10)